**Week 8 – Pair of lines**

(1)Find the angle between the pair of lines and separate them:

(i) $12x^2+7xy-12y^2=0$ (ii) $x^2+4xy+y^2=0$ (iii) $x^2-4y^2=0$

(iv) $x^2+2xy+y^2=0$ (v) $2x^2-7xy+3y^2=0$ (vi) $x^2+2xy+8y^2=0$

(vii) $x^2-2xy-3y^2=0$ (viii) $2x^2+5xy-3y^2=0$ (ix) $x^2-xy+3y^2=0$

(2)Show that the equation represents pair of lines and find their point of intersection:

(i) $3x^2+5xy-12y^2-19x+34y-14=0$ (ii) $x^2+4xy+4y^2+3x+6y+2=0$

(iii) $6x^2+xy-12y^2-14x+47y-40=0$ (iv) $6x^2+xy-y^2+4x-8y-16=0$

(v) $6x^2+2xy-20y^2-9x-7y+3=0$ (vi) $8x^2+12xy-8y^2-10x+10y-3=0$

(vii) $2x^2+3xy+y^2-x-1=0$ (viii) $2x^2+xy-y^2-3x+3y-2=0$

(ix) $2x^2-xy-y^2-4x-5y-6=0$ (x) $x^2-2xy-3y^2-2x+6y=0$

(3)Find the value of the constant such that the equation represents pair of lines:

(i) $x^2-3xy+2y^2+x-y+k=0$

(ii) $2x^2+3xy+by^2-x-1=0$

(4)Show that the following pair of lines form a square and find its area:

$12x^2+7xy-12y^2=0, \quad 12x^2+7xy-12y^2-x+7y-1=0$

**Week 9 – Circle and Parabola**

(1)Write the equation of the circle in Cartesian form and parametric form where:

(i)the center is (2, 3) and radius is 3 (ii)the center is (– 2, 1) and the radius is 2

(iii)the center is (3, – 2) and radius is 4 (iv)the center is (0, 2) and the radius is 5

(2)Find the equation of the circle in which the following points are ends of diameter:

(i) (1, 0), (3, 4) (ii) (0, 0), (– 2, 2) (iii) (4, 1), (2, 3)

(3)Write the equation of the tangent to each of the following circles at the given point:

(i) $x^2+y^2+8x-4y-41=0$ at (2, 7) (ii) $x^2+y^2-4=0$ at (2, 0)

(iii) $x^2+y^2-3x+4y-31=0$ at (– 2, 3) (iv) $x^2+y^2-2x-4y+3=0$ at (2, 3)

(4)Find the radical axis and the points of intersection of each pair of the circles:

(i) $x^2+y^2+3x-2y-4=0$, $x^2+y^2-2x-y-6=0$

(ii) $x^2+y^2-3x+4y-31=0$, $x^2+y^2-2x-4y+3=0$

(iii) $x^2+y^2+6x-2y-1=0$, $x^2+y^2+2x-y-8=0$

(iv) $x^2+y^2-4x+2y-3=0$, $x^2+y^2-6x-8y+3=0$

(5)Show that the two circles are orthogonal in the following:

(i) $x^2+y^2+6x+2=0$, $x^2+y^2+y-2=0$

(ii) $x^2+y^2+6x-2y+1=0$, $x^2+y^2-2x-6y-1=0$

(6)Determine the vertex, focus and the directrix of the following parabolas:

(i) $y^2=-8x$ (ii) $y^2-8x+2y+1=0$

(iii) $x^2=-12y$ (iv) $x^2-8x+8y-8=0$

(v) $y^2-8x+16=0$ (vi) $x^2=-8y+24$

(vii) $y^2+3x+4y+13=0$ (viii) $x^2-2x-8y-17=0$

(ix) $y^2+8x-4y-20=0$ (x) $y^2+4x+6y+1=0$

(xi) $x^2+4x+8y-28=0$ (xii) $x^2+6x+4y+9=0$

(7)Write the equation of the parabola in the following:

(i)Focus at the point (2, 0) and directrix is x + 2 = 0

(ii)Focus at the point (4, 0) and directrix is x = 0

(iii)Focus at the point (– 2, 0) and directrix is x – 4 = 0

(iv)Focus at the point (0, 3) and directrix is y + 4 = 0

(v)Focus at the point (3,– 6) and directrix is y – 8 = 0

(vi)Focus at the point (2, 4) and directrix is x – 10 = 0

(8)Write the equation of the parabola in the following:

(i)Focus at the point (3, 0) and directrix is x – y + 1 = 0.

(ii)Focus at the point (0, 2) and directrix is x – y – 2 = 0.

(iii)Focus at the point (1, –2) and directrix is x – y = 0.

**Week 10 – Ellipse and Hyperbola**

(1)Determine the center, vertices, foci, the major and minor axes of the ellipses:

(i) $\frac{x^2}{144}+\frac{y^2}{16}=1$ (ii) $\frac{x^2}{9}+\frac{y^2}{16}=1$ (iii) $\frac{x^2}{9}+\frac{(y-3)^2}{16}=1$

(iv) $x^2+4y^2+4x-24y+36=0$ (v) $16x^2+9y^2+32x-18y+25=0$

(vi) $x^2+4y^2-4x-8y+4=0$ (vii) $x^2+9y^2-6x+18y+9=0$

(viii) $x^2+4y^2+6x-16y+21=0$ (ix) $4x^2+5y^2-24x-30y+61=0$

(x) $9x^2+4y^2-54x+24y+81=0$ (xi) $9x^2+y^2-36x+6y+36=0$

(2)Write the equation of the ellipse in Cartesian form and parametric form where:

(i)Foci (4, 0), (– 4, 0) and the major axis is 10.

(ii)Foci (4, 0) (– 4, 0) and the minor axis is 10.

(iii)Foci (– 10, 0), (– 2, 0) and the major axis is 10.

(iv)Foci (0, 3), (0, – 3) and the major axis is 10.

(v) Foci (6, 3), (6, – 3) and the major axis is 10.

(vi) Foci (6, 3), (– 2, 3) and the major axis is 12.

(vii)Vertices (5, 0), (– 5, 0) and foci (3, 0), (– 3, 0)

(viii)Vertices (5, 2), (– 5, 2) and foci (3, 2), (– 3, 2)

(3)Write each the equation of each ellipse in parametric form:

(i) $2x^2+3y^2+8x+12y+8=0$

(ii) $9x^2+y^2-36x+6y+36=0$

(4)Determine the center, vertices, foci, the transverse and conjugate axes of the following hyperbolas:

(i) $\frac{x^2}{144}-\frac{y^2}{16}=1$ (ii) $\frac{x^2}{9}-\frac{y^2}{16}=1$ (iii) $\frac{x^2}{9}-\frac{(y-3)^2}{16}=1$

(iv) $x^2-4y^2+4x+24y-36=0$ (v) $16x^2-9y^2+32x+18y+7=0$

(vi) $9x^2-4y^2-16y+20=0$ (vii) $x^2-y^2-2x+4=0=0$

(vii) $4x^2-9y^2-8x+36y+4=0$ (ix) $2x^2-y^2-12x+6y-13=0$

(x) $4x^2-y^2+16x-4y+16=0$ (xi) $4x^2-y^2+16x-4y-16=0$

(5)Write the equation of the hyperbola in the following:

(i)Foci (4, 0), (– 4, 0) and the transverse axis is 4.

(ii)Foci (4, 0), (– 4, 0) and the conjugate axis is 2.

(iii)Foci (– 10, 0), (– 2, 0) and the transverse axis is 6.

(iv)Foci (0, 5), (0, – 5) and the transverse axis is 8.

(v) Foci (6, 3), (6, – 3) and the transverse axis is 4.

(vi) Foci (6, 3), (– 2, 3) and the transverse axis is 4.

(vii)Vertices (5, 0), (– 5, 0) and foci (8, 0), (– 8, 0)

(viii)Vertices (5, 2), (– 5, 2) and foci (7, 2), (– 7, 2)

(6)Determine the type of each curve in the following:

(i) $5x^2 - 4xy + 2y^2 - 6 = 0$ (ii) $2xy + 4x - 8y - 17 = 0$

(iii) $24xy - 7y^2 - 120y - 144 = 0$ (iv) $8x^2 - 4xy + 5y^2 - 36x + 18y + 9 = 0$

(v) $16x^2 - 24xy + 9y^2 - 94x + 8y - 99 = 0$ (vi) $9x^2 + 24xy + 16y^2 - 3x - 4y - 6 = 0$

(vii) $3x^2 + 8xy - 3y^2 + 54x + 22y - 77 = 0$ (viii) $3x^2 - 8xy - 3y^2 - 2x - 4y - 1 = 0$

(ix) $x^2 - 2xy + y^2 - 6x - 2y + 1 = 0$ (x) $4x^2 + 6xy - 4y^2 - 6x + 8y - 4 = 0$

## Week 11 – Solid Geometry

(1)Write the equation of the plane which satisfies the conditions:

(i) passes through the points A(1, 0, 1), B(2, 1, 0), C(0, 2, 1).

(ii)passes through the points A(0, 0, 0), B(1, 1, 0), C(1, 1, 1).

(iii)passes through the point A(1, 0, 1) and its perpendicular vector $\overline{U} = 2i + 2j + 3k$

(iv)passes through the point A(1, 0, 1) and parallels to the plane 2x + y – 3z + 2 = 0

(v)passes through the point A(2, 1, 1) and perpendicular to the planes:

2x + y – 3z + 2 = 0 and x + y + 2x = 0

(2)Find the symmetric form and parametric form of each line:

(i) It passes through the point (2, 0, –1) and parallels to the vector $\overline{U} = 2\bar{i} + 2\bar{j} - \bar{k}$

(ii)It passes through the point (3, 0, –2) and parallels to the vector $\overline{U} = \bar{i} + 2\bar{j} - 3\bar{k}$

(iii) It passes through the points (3, 0, –3) and (0, 3, 1)

(iv) It passes through the points (1, 2, –1) and (2, 3, 4)

(3)Find the angle between each pair of lines:

(i) $\frac{x-4}{4}=\frac{y-1}{0}=\frac{z}{3}$ and $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z}{2}$

(ii) $\frac{x}{-2}=\frac{y-1}{1}=\frac{z+2}{2}$ and $\frac{x-2}{2}=\frac{y-1}{1}=\frac{z+3}{-2}$

(iii) $x=3t+4,\ \ y=2,\ \ z=4t-1,\quad t\ in\ R$

$x=t+2,\ \ y=2t+1,\ \ z=2t-1,\quad t\ in\ R$

(iv) $x=2t+1,\ \ y=2t-1,\ \ z=t,\quad t\ in\ R$

$x=t+2,\ \ y=2t+1,\ \ z=2t-1,\quad t\ in\ R$

(4)Find the angle between the two planes :

(i) 2x – y + 2z + 1 = 0, x + 2y + 2z = 0

(ii) 2x + 3y – z + 1 = 0, x – y + z + 3 = 0

(iii) x + y + z + 2 = 0, 2x + 2y + z = 0

(iv) 4x + 3y – 3 = 0, x – 2y + 2z + 5 = 0

(5)Find the angle between the plane and the line, also, obtain the point of intersection of them:

(i) 2x – y + 2z + 1 = 0 and $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z+1}{-2}$

(ii) x + 2y + 2z = 0 and $\frac{x+1}{3}=\frac{y}{4}=\frac{z-3}{0}$

(iii) 2x + 3y + 3z – 8 = 0 and $\frac{x+1}{3}=\frac{y-3}{-1}=\frac{z+3}{2}$

(iv) x – 2y + 2z - 3 = 0 and $\frac{x}{3}=\frac{y-2}{1}=\frac{z-1}{1}$

(6)Show that the lines are skew :

(i) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{-1}$ and $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z-1}{1}$

(ii) $\frac{x}{1}=\frac{y-2}{2}=\frac{z-1}{2}$ and $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{2}$

(7)Write the equation of the sphere which satisfies the conditions:

(i) the center is (1, 0, 1) and radius is 3.

(ii) the center is (1, 2, – 3) and radius is 4.

(ii) The center is the point (1, 1, –2) and radius is 2/3.

(8)Determine the center and radius of each sphere:

(1) $x^2+y^2+z^2-6x-2y+6=0$ (2) $x^2+y^2+z^2-4z-1=0$

(3) $x^2+y^2+z^2-2x+4y-4z+9=0$ (4) $x^2+y^2+z^2-2x+2y-2=0$

(5) $x^2+y^2+z^2-2x-6z+2=0$ (6) $4+6x-5z-2x^2-2y^2-2z^2=0$

(9)Write the name of each in the following :

(i) $x^2+y^2+z^2-6x-2y-1=0$ (ii) $x^2+y^2+z^2-2x+4z-1=0$

(iii) $x^2+y^2+z^2-4z=0$ (iv) $x^2+4z^2-2=0$

(v) $2y^2+z^2-x^2=0$ (vi) $x^2+z^2-3=0$

(vii) $x^2+z^2-4y^2=0$ (viii) $x^2+3y^2=5$

(ix) $x^2+y^2+z^2-2=0$ (x) $x^2+y^2-3z^2=0$

(xi) $x^2-y^2+z^2=0$ (xi) $x^2+y^2-2z^2=0$

| **Name** | **Equation** |
|---|---|
| Straight Line | $ax + by + c = 0$, horizontal when $a = 0$, vertical when $b = 0$ |
| Pair of Lines | • $ax^2+2hxy+by^2=0$, passes through (0, 0), $\tan\theta=\pm 2\dfrac{\sqrt{h^2-ab}}{a+b}$ <br> • $ax^2+2hxy+by^2+2gx+2fy+c=0$ if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$ |
| Circle | $x^2+y^2+2gx+2fy+c=0$ Or $(x-x_1)^2+(y-y_1)^2=a^2$ <br> Center is $(x_1,y_1)$, radius is $a$ |
| Parabola | **Horizontal** $(y-y_1)^2=4a(x-x_1)$, vertex is $(x_1,y_1)$, focus is $(x_1+a,y_1)$ <br> **Vertical** $(x-x_1)^2=4a(y-y_1)$, vertex is $(x_1,y_1)$, focus is $(x_1,y_1+a)$ |
| Ellipse | $\dfrac{(x-x_1)^2}{a^2}+\dfrac{(y-y_1)^2}{b^2}=1$, center is $(x_1,y_1)$ <br> **Horizontal** when $a > b$, $a^2=b^2+c^2$ <br> Vertices are $V_1(x_1+a,y_1)$, $V_2(x_1-a,y_1)$ <br> Foci are $F_1(x_1+c,y_1)$, $F_2(x_1-c,y_1)$ <br> Ends of minor axis are $M_1(x_1,y_1+b)$, $M_2(x_1,y_1-b)$ <br> **Vertical** when $a < b$, $b^2=a^2+c^2$ <br> Vertices are $V_1(x_1,y_1+b)$, $V_2(x_1,y_1-b)$ <br> Foci are $F_1(x_1,y_1+c)$, $F_2(x_1,y_1-c)$ <br> Ends of minor axis are $M_1(x_1+a,y_1)$, $M_2(x_1-a,y_1)$ |
| Hyperbola | **Horizontal** $\dfrac{(x-x_1)^2}{a^2}-\dfrac{(y-y_1)^2}{b^2}=1$, center is $(x_1,y_1)$, $c^2=a^2+b^2$ <br> Vertices are $V_1(x_1+a,y_1)$, $V_2(x_1-a,y_1)$ <br> Foci are $F_1(x_1+c,y_1)$, $F_2(x_1-c,y_1)$ <br> Ends of conjugate axis are $M_1(x_1,y_1+b)$, $M_2(x_1,y_1-b)$ <br> **Vertical** $\dfrac{(y-y_1)^2}{b^2}-\dfrac{(x-x_1)^2}{a^2}=1$, center is $(x_1,y_1)$, $c^2=a^2+b^2$ <br> Vertices are $V_1(x_1,y_1+b)$, $V_2(x_1,y_1-b)$ <br> Foci are $F_1(x_1,y_1+c)$, $F_2(x_1,y_1-c)$ <br> Ends of conjugate axis are $M_1(x_1+a,y_1)$, $M_2(x_1-a,y_1)$ |

| | |
|---|---|
| General Equation | $ax^2+2hxy+by^2+2gx+2fy+c=0$<br>This equation represents:<br>(1)Pair of lines if $\Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$<br>(2) If $\Delta\neq 0$<br>2-1 Circle if h = 0, a = b<br>2-2 Parabola if $h^2=ab$<br>2-3 Ellipse if $h^2<ab$<br>2-4 Hyperbola if $h^2>ab$ |
| Plane | ax + by + cz + d = 0, normal vector is $\overline{N}=a\bar{i}+b\bar{j}+c\bar{k}$ |
| Line in Space | $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$, passes through $(x_1,y_1,z_1)$ and // $\overline{N}=a\bar{i}+b\bar{j}+c\bar{k}$ |
| Sphere | $x^2+y^2+z^2+c_1x+c_2y+c_3z+c_4=0$<br>Or $(x-x_1)^2+(y-y_1)^2+(z-z_1)^2=a^2$, center is $(x_1,y_1,z_1)$, radius is a |
| Ellipsoid | $c_1x^2+c_2y^2+c_3z^2+c_4x+c_5y+c_6z+c_7=0$<br>Or $\frac{(x-x_1)^2}{a^2}+\frac{(y-y_1)^2}{b^2}+\frac{(z-z_1)^2}{c^2}=1$,<br>Center is $(x_1,y_1,z_1)$, axes are a, b, c |
| Cone | $(x-x_1)^2+(y-y_1)^2=a(z-z_1)^2$, vertex is $(x_1,y_1,z_1)$, axis is // z – axis<br>$(x-x_1)^2+(z-z_1)^2=a(y-y_1)^2$, vertex is $(x_1,y_1,z_1)$, axis is // y – axis<br>$(y-y_1)^2+(z-z_1)^2=a(x-z_1)^2$, vertex is $(x_1,y_1,z_1)$, axis is // x – axis |
| Paraboloid | $(x-x_1)^2+(y-y_1)^2=a(z-z_1)$, vertex is $(x_1,y_1,z_1)$, axis is // z – axis<br>$(x-x_1)^2+(z-z_1)^2=a(y-y_1)$, vertex is $(x_1,y_1,z_1)$, axis is // y – axis<br>$(y-y_1)^2+(z-z_1)^2=a(z-z_1)$, vertex is $(x_1,y_1,z_1)$, axis is // x – axis |
| Cylinder | $(x-x_1)^2+(y-y_1)^2=a^2$, axis is // z – axis, passes through $(x_1,y_1,0)$<br>$(x-x_1)^2+(z-z_1)^2=a^2$, axis is // y – axis, passes through $(x_1,0,z_1)$<br>$(y-y_1)^2+(z-z_1)^2=a^2$, axis is // x – axis, passes through $(0,y_1,z_1)$ |

| Engineering Mathematics and Physics Department<br>Mathematics 2 Code: Math 102<br>Time Allowed: **2 hours** | **Faculty of Engineering** | Academic year: 2010/2011<br>Semester: Spring<br>Final Exam: June 2011<br>Examiner: Dr. Mona Samir<br>Dr. Mohamed Eid |
|---|---|---|
| **Answer 5 questions only** | | **Marks** |

| | Marks |
|---|---|
| **Question 1** | |
| (a)Using mathematical induction to prove the validity of the following: $\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\cdots+\frac{1}{(n+1).(n+2)}=\frac{n}{2.(n+2)}$. | 4 |
| (b) Use De Moiver's theorem to evaluate: $(2-3i)^{\frac{3}{4}}$. | 4 |
| **Question 2** | |
| (a) Find the sum $\sum_{r=1}^{n}\frac{1}{(r+1)(r+3)}$ | 4 |
| (b) Solve the following linear system by inverse method: $x-y+2z=6,\ 2x+y-z=5,\ x+y-z=2.$ | 4 |
| **Question 3** | |
| (a)Using the binomial theorem, expand $(5-3x)^{\frac{5}{3}}$. | 2 |
| (b)Find the eigenvalues and the eigenvectors of the matrix $A=\begin{bmatrix}2 & 4\\ 3 & 3\end{bmatrix}$. | 4 |
| (c)If α,β,γ,δ are the roots of the equation: $x^4-15x^2-10x+24=0$, then find (i) $\sum_i C_i^2$ (ii) $\sum_{i,j} C_i^2 C_j$ | 2 |
| **Question 4** | |
| (a)Complete the statement: The ellipse is locus of moving point such that…. | 2 |
| (b)Write the equation of parabola with focus F(0, 4) and direcrtix y = 0. | 2 |
| (c)Separate the lines $2x^2+3xy-2y^2-x+3y-1=0$. | 4 |
| **Question 5** | |
| (a)Write the equation of circle where (3, – 2), (1, 2) are ends of its diameter. | 2 |
| (b)Write the equation of plane passing through the point (2, – 1, 1) and parallels to $2x-y+z=0$ | 2 |
| (c)Find center, vertices and sketch the ellipse $9x^2+4y^2-24y=0$. | 4 |
| **Question 6** | |
| (a)Sketch the surfaces: (i) $x^2+y^2+z^2-2y=0$ (ii) $x^2+(y-1)^2-z^2=0$ | 4 |
| (b)Find center, vertices and sketch the hyperbola $x^2-y^2+4x+2y-1=0$ | 4 |

*Good luck* *Dr. Mona Samir* *Dr. Mohamed Eid*

| | | |
|---|---|---|
| Eng. Mathematics and Physics Department<br>Mathematics 2-Code: Math 102<br>Final Exam: 31 / 7 / 2011<br>Time Allowed: **2 hours** | Modern University For Technology & Information | Academic year: 2010 / 2011<br>Semester: Summer<br>Examiner: Dr. Mona Samir<br>Dr. Mohamed Eid |
| **Answer 5 questions only** | **Faculty of Engineering** | **Marks** |

## Question 1

(a)Using mathematical induction to prove the validity of the following: 4

$$\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\cdots+\frac{1}{(n+1).(n+2)}=\frac{n}{2(n+2)}.$$

(b)Find the eigenvalues and the eigenvectors of the matrix $A=\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}$. 4

## Question 2

(a)Solve the following linear system by inverse method:

$x+y+z=5,\quad 2x-y+z=2,\quad 2x+2y-z=4$ 5

(b) Using the binomial theorem, expand $\sqrt{5-2x^3}$. 3

## Question 3

(a) Use De Moiver's theorem to evaluate $(4-8i)^{\frac{5}{2}}$. 2

(b) Find the sum to n terms of the series: $\sum_{r=1}^{n}\frac{1}{(r+2)(r+3)}$ 3

(c )Solve the equation $x^3-12x+16=0$ if the number 2 is a repeated root. 3

## Question 4

(a)Complete the statement: The parabola is locus of moving point such that…. 2

(b)Write the equation of parabola with focus F(5, 0) and direcrtix x = 1. 2

(c)Separate the lines $2x^2+3xy-2y^2-x+3y-1=0$. 4

## Question 5

(a)Determine the center and radius of the circle $x^2+y^2-6x+8y=0$. 2

(b)Write the equation of plane passing through the points:

(2, – 1, 1), (1, 2, 1), (0, 3, 3). 3

(c)Find center, vertices and sketch the hyperbola $y^2-x^2+4x=0$. 3

## Question 6

(a)Sketch the surfaces: (i) $y^2-x^2-z^2=0$ (ii) $x^2+y^2+z^2-4y-5=0$ 2+2

(b)Find center, vertices and sketch the ellipse $9x^2+4y^2-36x=0$ 4

*Good luck* *Dr. Mona Samir* *Dr. Mohamed Eid*

| | | |
|---|---|---|
| Basic Science Department<br>Mathematics 2 Code: Math 102<br>Final Exam: 17 – 1 – 2013<br>Time Allowed: 2 hours | Modern University For Technology & Information | Academic year: 2012 / 2013<br>Semester: Autumn<br>Examiner: Dr. Mohamed Eid |
| **Answer All questions** | Faculty of Engineering | Total Marks 40 |

**Question 1**

(a) If $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & 4 \end{bmatrix}$ — 4

Find, if possible, A + B, A.B$^t$, A + B$^t$, A.B, |A.B|

(b) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$. — 4

(c) Determine the type of solution of the linear system: — 4

$$x + y + z = 5,\ x - y + z = 2,\ 2x + 2z = 7.$$

**Question 2**

(a) Using the binomial theorem, expand $\frac{1}{\sqrt{1-2x}}$ — 2

(b) Using mathematical induction, prove that: — 3

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n.(n+1)} = \frac{n}{(n+1)}$$

(c) If $z_1 = 2 - 3i$, $z_2 = -1 + 2i$. Find $z_1 . z_2$, $(z_1 + z_2)^{10}$. — 3

**Question 3**

(a)State the definition of parabola. — 2

(b)Separate the lines $x^2 + xy - 2y^2 + 3x + 6y = 0$. — 3

Also, find the angle between them.

(c) Write the equation of circle with center (2, – 3) and radius 3/2. — 2

(d)Determine the vertex, focus and sketch the parabola $x^2 - 4x + 8y - 4 = 0$. — 3

**Question 4**

(a)Find center, vertices and sketch the ellipse $x^2 + 4y^2 + 4x + 8y + 4 = 0$. — 4

(b)Describe the surface $x^2 + y^2 + z^2 - 2x + 4y = 0$ — 3

(c)Write the equation of plane that passes through (1, 2, 3), (2, 1, 1), (3, 0, 2). — 3

**Good luck** **Dr. Mohamed Eid**

| Basic Science Department<br>Math. 2 Code: Math 102<br>Final Exam: 26 – 5 – 2013<br>Time Allowed: 2 hours | Modern University For Technology & Information | Academic year: 2012 / 2013<br>Semester: Spring<br>Examiner: Dr. Mona Samir<br>Dr. Mohamed Eid |
|---|---|---|
| **Answer All questions** | Faculty of Engineering | Total Mark: 40 |

**Question 1**

(a) If $\alpha, \beta$ *and* $\gamma$ are the roots of the equation: $x^3 - 6x - 3x^2 + 8 = 0$,

Find: (i) $\sum_{i=1}^{3} C_i^2$ (ii) $\sum_{i=1}^{3} C_i^3$ (iii)The roots if they form an A.S. — 4

(b) Using mathematical induction, prove that:

$$\frac{1}{2x3} + \frac{1}{3x4} + \frac{1}{4x5} + \cdots + \frac{1}{(n+1)(n+2)} = \frac{n}{2(n+2)}$$ — 3

(c)Find the sum to **n** terms of the series: $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \cdots + \frac{1}{n(n+1)}$ — 3

**Question 2**

(a) Find the eigenvalues and the eigenvectors of the matrix: $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ — 4

(b) Solve the equation $x^3 - 8x^2 + 21x - 20$, if $2 - i$ is one of the root. — 3

(c) Solve the following linear system by inverse method: — 3

$$y + 2z + 2x - 8 = 0, \quad x + z - y = 1, \quad x + 2z + y = 7.$$

**Question 3**

(a)State the definition of parabola. — 2

(b) Determine the center and radius of the circle $x^2 + y^2 + 4x - 6y + 3 = 0$. — 4

Also, write its tangent at the point (1, 2).

(c) Find center, vertices and sketch the hyperbola $4x^2 - y^2 + 24x + 4y + 36 = 0$. — 4

**Question 4**

(a)Find center, vertices and sketch the ellipse $x^2 + 4y^2 + 4x + 8y + 4 = 0$. — 3

(b)Write the equation of plane that passes through (1, 2, 3), (2, 0, 1), (4, 1, – 1). — 3

(c)Find the angle between the line $\frac{x-2}{2} = \frac{y-1}{1} = \frac{z}{-1}$ and the plane $x - 2y + z + 1 = 0$ — 4

Also, find the point of intersection.

*Good luck* *Dr. Mona Samir* *Dr. Mohamed Eid*

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| Basic Science Department<br>Mathematics II Code: Math 102<br>Final Exam: May 2014<br>Time Allowed: **2** Hours | Modern University For Technology & Information | Academic year: 2013 / 2014<br>Semester: Spring<br>Examiners: Dr. Mona Samir<br>Dr. Mohamed Eid |
| Answer **All** questions | Faculty of Engineering | No. of Questions: 4 Total Mark: 40 |

**ممنوع** إستخدام المحمول كآلة حاسبة. يُسمح **فقط** بإستخدام الألة الحاسبة العادية

Do not use Mobile as Calculator. Only use regular Calculator

**Question 1**

(a) Prove using mathematical induction that for all $n \geq 1$,

$1 + 4 + 7 + \cdots + (3n-2) = \frac{1}{2}n(3n-1)$ 3

(b) Find the sum of **n** terms of the series: $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ 2

(c) Using Horner's method, divide $(x^4 - 6x^3 + 28x - 30)$ by $(x - 5)$. 2

(d) If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of the equation: $2x^4 - 4x^3 - 12x^2 + 16 = 0$.

Then find : (i) $\sum_{i=1}^{4} (\alpha_i)^2$ (ii) the roots if they form geometric series. 3

**Question 2**

(a) Evaluate $(-4 - 8i)^{2/3}$ 2

(b)Find the eigenvalues and the eigenvectors of the matrix A= $\begin{bmatrix} -5 & 0 & 0 \\ 3 & 7 & 0 \\ 4 & -2 & 3 \end{bmatrix}$. 4

(c)Solve the following linear system using inverse matrix: 4

$-y - z + 2x = 4, \quad 4y - 2z - 11 = -3x, \quad 4z - 2y + 3x = 11.$

**Question 3**

(a)State the definition of the parabola. 2

(b) Show that the circles $x^2 + y^2 + 4x + 2y - 3 = 0$, $x^2 + y^2 - 6x + 6y - 3 = 0$ 2

(c)Write the equation of circle where the points (2, 1), (0, –3) are ends of diameter. 3

Also, find the equation of tangent at the point (2, 1).

(d)Find center, vertices and sketch the ellipse $4x^2 + y^2 - 8x + 4y + 4 = 0$. 3

**Question 4**

(a)Write the equation of the sphere of radius 2 and its center at the point (1, 2, – 2). 2

(b)Describe the surface $2x^2 - y^2 - 3z^2 = 0$. 2

(c)Write the equation of the plane that passes through the point (1, –1, 3) and 3

parallels to the plane 2x + y – 2z = 3.

(d)Determine the value of c such that the following equation represents pair of 3

lines: $2x^2 + 3xy + y^2 - x + c = 0$.

**Good luck** **Dr. Mona Samir** **Dr. Mohamed Eid**